Phase Distribution and Residence Time in a Spouted Bed

Particle movement in the core and in the annulus of a semicircular, vertical, spouted bed was examined by high-speed tracer photography. Use of a functional analogy with a free jet establishes an equation correlating core air velocity and relative bed height. Dependence of bed void fraction and particle throughput vs. relative bed height shows good agreement with previous work. Particle mean retention time and air flow distribution in the annulus and the spout were obtained by calculational estimates.

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SCOPE

The efficiency of the spouted bed in its main practical applications, as a dryer and reactor, is dependent to a large extent upon the flow characteristics of the particles in the bed and in the spout. Although the spouted bed has received considerable attention in the last years, (Mathur and Epstein, 1974) data on flow and residence time distribution are particularly scarce. The objective of

this work is to provide laboratory scale data and correlations concerning air and particle flow distribution in the annulus and spout, particle voidage in the spout, and particle residence time as functions of bed characteristics. Axial and radial particle movements in the bed were examined and velocities were measured from high-speed tracer photographs.

CONCLUSIONS AND SIGNIFICANCE

From application of Ergun's equation on the expanded bed, the air velocity in the annulus and in the spout was calculated with results in agreement with those of previous researchers (Mathur and Gishler, 1955; Thorley et al., 1959). A correlation of air velocity variation in the spout vs. bed height was developed [Equation (1)].

Particle velocity estimates from the examination of high-speed photographs and simplifying assumptions re-

garding the state of the bed have led to the calculation of the bulk density and voidage of both annulus and spout. By graphical and analytical integration [Equation (11)] it was possible to obtain the average retention time of particles in each characteristic zone and, by summation, the total retention time in the system. Particle recirculation ratio was estimated by a simplified form of Craya-Curtet's equation.

EXPERIMENTAL PROCEDURE

The data reported deal with spouted beds (initial heights H=130 and 230 mm) obtained by injecting air through a 4 mm orifice placed at a conical apex (60 deg cone angle) of a vertical, semicircular column ($D_c=100$ mm) provided with a transparent frontal wall. Four types of particles were used having different geometrical dimensions ($d_p=1,865$ to 3,430 mm), shapes (spherical, cylindrical and parallelepipedical), sphericities ($\psi=0,65$ to 1,0), and densities ($\rho_s=0,949$ to 1,309 kg/dm³) (Patrascu, 1974).

Colored particles were used as tracers, and their displacement was observed and measured by high-speed photography. Slides were taken at frequencies of 500 s⁻¹ for the annulus and of 1700 s⁻¹ for the spout at air mass velocities (based on the cross section of the empty column) in the range of 0,677 and 1,031 kg/m²s, exceeding by up to 20% the minimum velocity required to maintain a stable spout. Particle velocities in the annulus are of the order of centimeters per second, while in the core they may reach a few meters per second.

Particle trajectories in the core and in the annulus are presented in Figure 1.

Owing to the high speed of the particles in the cone and to blurring of the spout-annulus boundary at the top end of the bed, no satisfactory photographs were obtained in these sections, even at 1 700 s⁻¹.

DISCUSSION

Air Distribution in the Spout and Annulus

As was done by Mathur and Gishler (1955), Thorley et al. (1959), Lim and Mathur (1974), and Van Velzen et al. (1974), annular zone air throughput was calculated from static pressure drop measurements at the wall along the cylindrical part of the column.

Since gas flow in the annulus is laminar ($Re_p < 15$), it may be assumed to be of a pattern similar to that of a fixed bed in which pressure drop is proportional to gas velocity. This would justify the calculation of gas velocities at various column heights by Ergun's equation and of the corresponding air flow rates from known annulus cross sections. By difference from the total

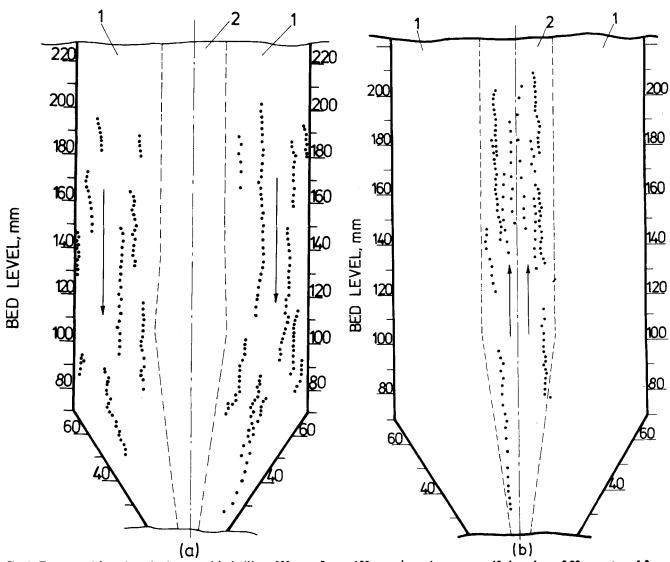


Fig. 1. Tracer particle trajectories in spouted beds (H=230 mm; $D_c=100$ mm; $d_o=4$ mm; $\varphi=60$ deg; $d_p=2,29$ mm; $\psi=1,0$; $\rho_s=1,013$ kg/dm³). 1—Annulus; 2—spout. (a) w=1,27 kg/m²s; 500 slides/s. (b) w=1,31 kg/m²s; 1 700 slides/s.

air throughput, the corresponding air flow rates and air velocities in the core were arrived at for each given bed height. In all calculations, the spout diameter for the entire cylindrical height of the vessel was assumed to be constant and equal to that measured at the top one third of the bed. Air rate variation in the annulus vs. relative bed height is presented in Table 1 and Figure 2.

Visual observations and the photos taken show that there is apparently no fluidization taking place in the annulus. At the low particle velocities, measured in our experiments, of the order of a few centimeters per second, the wall effect has probably no appreciable influence on the radial pressure gradient. The data agree with findings of previous researchers (Mathur and Gishler, 1955; Thorley et al., 1959).

As expected, the annulus relative air rate increases from the base to the upper part of the bed, and at the exit from the conus (in the cylindrical zone) approximately 20% of the total air stream passes through the annular zone for the H=130 mm bed and approximately 35% for the one with H=230 mm.

It may be noted that for a given particle-column system $(D_c, d_o, T, d_p \text{ constant})$, an increase in the initial bed height H is associated with an increase in the relative air flow through the annulus as shown by data from

Table 1 and Figure 2. For the H=230 mm bed, the relative air rate in the annulus is in agreement with previous data of Mathur and Gishler (1955) and Thorley et al. (1959). For shorter beds (H=130 mm) of comparable heights, however, the relative air throughput at the upper end of the annulus exceeds the value reported by Muhlenov and Gorstein (1964). Apparatusgeometry differences may account for these discrepancies.

In agreement with findings of Thorley et al. (1959), an increase in the total air rate above that required for maintaining a stable spout shows no appreciable influence on the relative amount of the air passing through the annulus.

Air velocity variation in the spout along the cylindrical part of the bed vs. relative bed height (h/H) is presented in Figure 3. In all cases, air velocity undergoes a drop with increasing relative bed height. It is also to be noted that for both initial bed heights (H=230 and 130 mm) an increase of the total air rate is followed by an increase of air and particle velocity in the spout and a corresponding reduction of particle residence time.

Relative Air Velocity in the Spout

The mechanistic analogy of the spouted bed with a free jet of gas suggests similar formal relations:

Table 1. Air Distribution in the Annulus of the Spouted Bed (Particles: $d_p=1,865$ mm; $\rho_8=1,309$ kg/dm³; $\psi=1,0$)

	$\frac{h}{H}$	% air follow	Annulus air				
H, mm	(cylidr. zone)	0,677*	0,685	0,838			throughput, V_4 , Nm $^3/h$
130	0,539	27,3	27,0	21,8			2,00-2,02
	0,576	29,7	29,4	23,8			2,18-2,20
	0,615	32,2	31,8	25,8			2,36-2,38
	0,654	35,4	35,0	28,3			2,59-2,62
	0,692	37,8	37,4	30,3			2,77-2,80
	0,731	42,4	41,8	33,7			3,09-3,13
	0,770	43,0	42,5	34,4			3,14-3,18
	0,808	45,7	45,1	36,5			3,34-3,38
	0,846	47,5	47,0	38,0			3,48-3,52
	0,885	50,0	49,4	40,1			3,66-3,70
	0,924	52,5	51,8	42,0			3,84-3,88
	0,963	55,4	54,7	44,2			4,05-4,10
		0,855°	0,864	0,948	0,979	1,031	
230	0,304	40,8	40,3	37,0	35,7	33,8	3,82-3,85
	0,348	43,6	43,1	39,6	38,2	36,1	4,08-4,11
	0,391	47,9	47,4	43,3	42,0	39,6	4,49-4,52
	0,435	50,7	50,1	46,0	44,4	42,0	4,75-4,79
	0,479	53,0	52,4	48,1	46,4	44,9	4,96-5,00
	0,521	55,0	54,5	50,0	48,1	45,6	5 ,15- 5 ,2 0
	0,565	57,4	56,6	51,9	50,1	47,5	5,36-5,41
	0,609	58,4	57,7	53,1	51,2	48,5	5,48-5,51
	0,652	60,1	59,5	54,5	52,9	50,0	5,65-5,68
	0,696	61,6	60,9	56,0	54,0	51,1	5,78-5,81
	0,739	62,7	62,0	56,9	54,6	51,7	5,85-5,90
	0,783	64,5	63,8	58,5	56,5	53,4	6,04-6,09
	0.826	65,4	64,6	59,4	57,4	54,3	6,14-6,17
	0,870	66,7	66,0	60,5	5 8,5	55,2	6,25-6,30
	0,913	68,0	67,1	61,6	59,5	56,4	6,36-6,41
	0,957	68,6	67,7	62,1	60,0	56,7	6,50-6,55

[•] Figures represent minimum velocities required to maintain a stable spout.

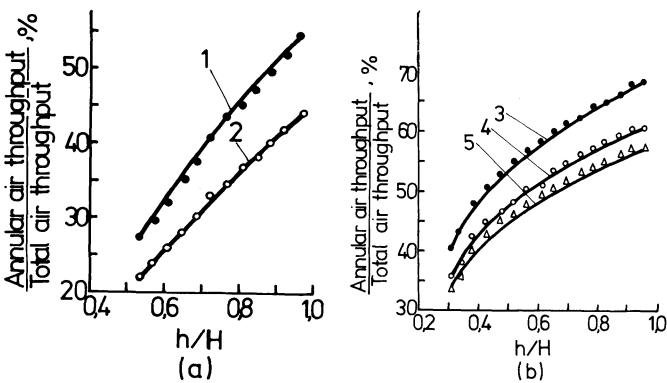
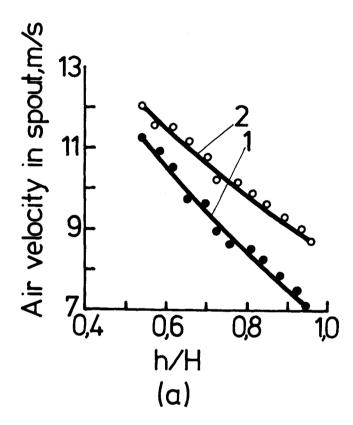


Fig. 2. Relative air throughput in the annulus (particle: $d_p=1,865\,$ mm; $\rho_s=1,309\,$ kg/dm³; $\psi=1,0$). Initial bed height, mm: a—130; b—230. Air mass velocity (based on empty column), kg/m²s: 1—0,685, 2—0,838; 3—0,864; 4—0,979; 5—1,031.



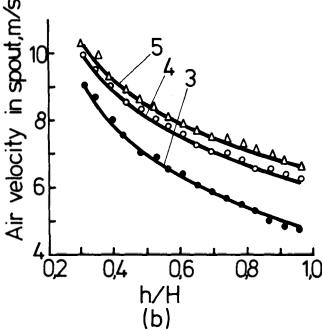


Fig. 3. Air velocity in the cylindrical part of the core vs. h/H (particle: $d_p=1,865$ mm; $\rho_s=1,309$ kg/dm³; $\psi=1,0$). Initial bed height, mm: a—130; b—230. Ratio w/w $_m$: 1—1,012; 2—1,238; 3—1,010; 4—1,110; 5—1,140.

$$v_f/v_{pc} = m \exp[-n(h/H)] \tag{1}$$

values of v_f/v_{pc} calculated by Equation (1) for H=230 mm from data given (m and n) in Table 2 are plotted in Figure 4, where numbers of curves are reference numbers of Table 2.

Since m and n depend on the total air mass velocity, the following correlations were presumed:

$$m = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4 \tag{2}$$

$$n = b_0 + b_1 w + b_2 w^2 + b_3 w^3 + b_4 w^4 \tag{3}$$

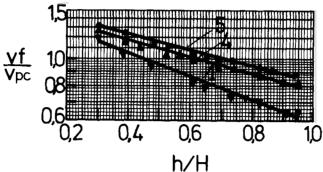


Fig. 4. Axial variation of v_f/v_{pc} for spouted beds (H=230 mm) vs. h/H.

Table 2. Constants m and n in Equation (1)

Ref. No.	Total air throughput, Nm³/h	air mass velocity (column), kg/m²s	$\frac{w}{w_m}$	m	n
1	9,34	0,855	1,00	1,547	0,970
2	9,45	0,864	1,01	1,548	0,930
3	10,35	0,948	1,11	1,557	0,735
4	10,70	0,979	1,14	1,565	0,700
5	11,30	1,031	1,21	1,589	0,669

Coefficients a_0 . . . a_4 and b_0 . . . b_4 were obtained by regression analysis.

The corresponding explicit relations

$$m = 3,070 - 0,802w - 5,879w^2 + 7,718w^3 - 2,534w^4$$

$$n = 2,387 + 13,703w - 43,845w^2 + 40,505w^3 - 12,063w^4$$
 (3a)

are plotted in Figure 5.

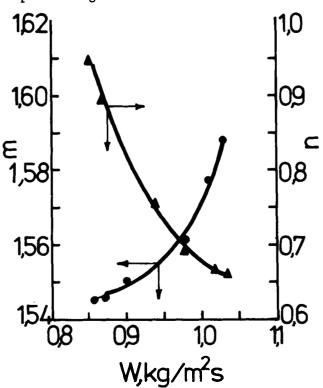


Fig. 5. Coefficients *m* and *n* of Equation (1) vs. total air mass velocity.

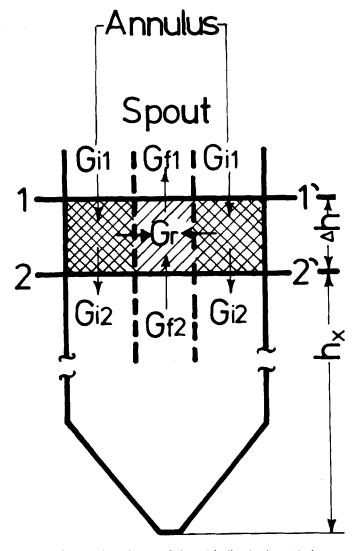


Fig. 6. Schematical axial and radial particle flow in the vertical section of a spouted bed.

Spout Particle Distribution and Voidage

Particle velocity determinations by tracer photography in known cross sections were used for calculating particle flow rates in the spout and in the annulus. With reference to Figure 6, assuming circumferential symmetry for both axial particle circulation and for the radial flow G_r along the spout, and taking $G_i = G_f$, we may write the following particle material balance equations:

$$G_r = G_{i1} - G_{i2} = G_{f1} - G_{f2} \tag{4}$$

Particle flow rate in the spout increases with increasing distance from the air orifice, leading to higher particle concentrations. Similar effect is produced by the particle velocity decrease from 2-2' to 1-1' as a result of the presence of a spout-to-annulus air slipstream.

By visual observations, it was established that in the annular zone, specifically in the cylindrical part of the vessel, up to $0.9 \ h/H$, bed density undergoes but a small variation, and for practical purposes it may be taken as constant. It was similarly observed that the annulus voidage is larger than that of an expanded fixed bed of particles and it is slightly smaller than that of a bed (containing similar particles) at the onset of fluidization. In this work, ϵ_i was assumed equal to ϵ

Bulk density in the annular zone is expressed by

$$\rho_{vi} = (1 - \epsilon)\rho_s \tag{5}$$

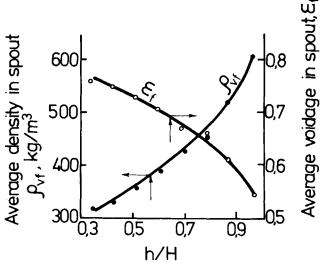


Fig. 7. Average bulk density (ρ_{vf}) and voidage (ϵ_f) in the spout vs. relative bed level $(d_p=1.865~\mathrm{mm};~\rho_s=1.309~\mathrm{kg/dm^3};~\psi=1.0;~w_m=0.855~\mathrm{kg/m^2s};~w=0.987~\mathrm{kg/m^2s}).$

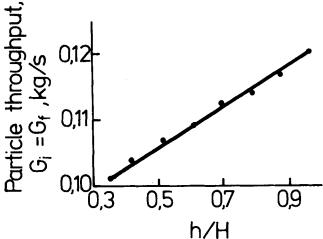


Fig. 8. Particle throughput vs. relative bed level (dp = 1,865 mm; ρ_s = 1,309 kg/dm³; ψ = 1,0; \mathbf{w}_m = 0,855 kg/m²s; \mathbf{w} = 0,987 kg/m²s).

Particle throughput in a cross section of the annular space may be obtained from known average particle velocity and annulus cross section

$$G_i = A_i v_{pi} \rho_{vi} \tag{6}$$

Since $G_f = G_i$, bulk density in the spout is expressed by

$$\rho_{vf} = G_i / A_f v_{pf} \tag{7}$$

and the voidage in the spout by

$$\epsilon_f = 1 - (\rho_{vf}/\rho_s) \tag{8}$$

Equations (5) to (8) were used to calculate for, each cylindrical bed, height increment of 0.02 m along the II = 230 mm bed average bulk density and voidage in the spout, and axial particle flow rate in both the spout and the annulus (see Figures 7 and 8).

Differences of 8% were noted between measured weight of particles in the spout and in the annulus and the values resulting from Equations (5) to (8). In the latter, for particle velocities in the annulus (v_{pi}) , weighted average values of the axial velocity at constant spout diameter were used (Suciu and Patrascu, 1975). It follows from the above that by assuming the annulus voidage to be equal to that of a fluidized bed of similar particles at

the onset of fluidization, it becomes possible by Equations (5) to (8) to calculate with reasonable accuracy bulk density and voidage for both the annulus and the spout.

Radial particle flow rate across the annulus-spout boundary was calculated (H = 230 mm) from particle flow variations along the column and is illustrated in Figure 9, where the percentages indicated are based on the particle stream falling from the outside spout into the bed. Although the total of these percentages increases with initial bed height, the average percentage of radial flow per unit height becomes smaller with increasing H.

Based on the above calculations, the variation of particle and air throughput ratio in the spout and in the annulus was calculated as a function of relative bed height. As shown by the data from Table 3, G_i (and G_f) increases with h/H, but the increase of G_{ai} is steeper and the ratio G_i/G_{ai} decreases. In the spout, however, an increase of h/H brings about a decrease of the air rate, the ratio G_f/G_{af} becoming larger.

Particle Residence Time

The time required for a particle to pass a whole cycle (cone, spout, disengaging zone, and annulus) is a statistical magnitude, characteristic for each particle. A total average cycle time for the particles may be obtained by the addition of the average retention times in each characteristic zone of the system:

$$\tau = \tau_i + \tau_c + \tau_f + \tau_{fe} \tag{9}$$

Within the experimental conditions of this work, the residence time in the annulus corresponds to a velocity of the order of centimeters per second, whereas in the spout it is of the order of meters per second.

The time required for an average particle to travel an elementary height dh of the spout may be expressed by

$$d\tau = dh/v_p \tag{10}$$

or

$$\tau = \int_{h_1}^{h_2} \frac{dh}{v_p} \tag{11}$$

whose integration requires the knowledge of the function relating v_p with bed height.

In the annular zone, $v_p = f(h)$ is linear along the cylindrical part of the vessel, and the average time τ_i may be obtained from

$$\tau_i = \frac{\Delta h}{\overline{v_n}} = \frac{h_1 - h_2}{\overline{v_n}} \tag{12}$$

Although measurements of particle velocities may be

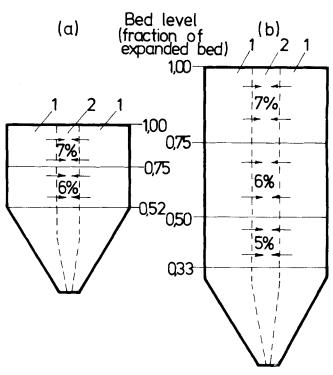


Fig. 9. Percentages of particle flow rate passing from the annulus into the spout along the cylindrical part of the vessel ($d_p = 1.865$ mm; $\rho_s = 1{,}309 \text{ kg/dm}^3$, $\psi = 1{,}0$).

Air mass velocity, kg/m²s: a—0,838; b—0,987. Minimum air mass velocity, kg/m²s: a-0,677; b-0,855.

strongly affected by wall effects, it was attempted to compensate for this by introducing in Equation (12) a weighted average particle velocity. For the H=230mm bed and $w = 0.987 \text{ kg/m}^2\text{s}$, a value of $\tau_i = 4.18 \text{ s}$ was obtained. Details on weighted average particle velocity calculations (v_p) are given elsewhere (Suciu and Patrascu, 1975). For the cylindrical height of the spout, where the particle velocity is a complex function, the retention time may be obtained by a graphical integration from a known particle velocity profile. By this procedure $\tau_f = 0.15$ s.

By a similar computational procedure for slightly different mass air velocities, w = 0.979 and 0.996 kg/m²s, mean retention times in the annulus of 4,51 and 3,97 s were found, respectively.

According to the foregoing, an increase of the total air throughput, above that required for maintaining a stable spout, has no important bearing on the gas flow

Table 3. Variation of G_i/G_{ai} and of G_f/G_{af} with Relative Bed Height, h/H

		Air in annulus			Air in spout		
$\frac{h}{H}$	$G_i = G_f$, kg/s	$\frac{\mathrm{Nm^3}}{s} \cdot 10^3$	$rac{ ext{kg}}{s} \cdot 10^3 \ (G_{ai})$	$\frac{G_i}{G_{ai}}$	$\frac{\mathrm{Nm}^3}{s} \cdot 10^3$	$rac{\mathrm{kg}}{s} \cdot 10^3 \ (G_{af})$	$rac{G_f}{G_{af}}$
0,956	0,1200	1,810	2,340	51,2	1,190	1,540	78,0
0,870	0,1171	1.740	2,250	52,0	1,235	1,600	73,2
0,782	0,1140	1.690	2,185	52,1	1,292	1,672	68,0
0,696	0.1127	1,610	2,080	54,1	1,365	1,766	63,7
0.609	0,1098	1,530	1,980	55,5	1,450	1,878	58,5
0,521	0,1069	1,438	1,860	57,5	1,540	1,992	5 3,6
0,435	0,1040	1,322	1,710	60,8	1,651	2,140	48,5
0,348	0,1012	1,140	1,477	68,5	1,840	2,380	42,5

rate through the annular zone, since the excess stream passes largely through the spout. Particle circulation in both zones is speeded up, however, by an increase of gas throughput.

Craya-Curtet Number for Spouted Beds

In an isothermal free jet of air discharging in a vessel of a cylindrical or parallelepipedical form, the extent recirculation may be expressed by the Craya-Curtet number (Hottel and Sarofim, 1967; Narashimhan, 1971):

$$Ct = \frac{1 + \frac{m_i}{m_o}}{\left[\left(\frac{m_i}{m_o}\right)^2 \left(\frac{i_o}{i_i}\right) - \frac{m_i}{m_o} - \frac{1}{2}\right]^{1/2}}$$
(13)

Taking into consideration that in a spouted bed the ratio $(m_i/m_o)^x$ (i_o/i_i) is considerably larger than the other terms and that m_i/m_o is larger than 1, and introducing i = mv, the above expression of Ct becomes

$$Ct = \sqrt{\frac{m_i}{m_o} \frac{v_i}{v_o}} \tag{14}$$

For H = 230 mm bed, which is approximately 50%of the maximum height of a spoutable bed with the same particles, the value of Ct calculated by Equations (13) and (14) was 0.0592 and 0.0575, respectively. For gases discharging into a mass of gas, recirculation is practically complete at Ct = 0, while at Ct = 0.74 there is practically no recirculation. For higher beds, approaching the maximum height of a spoutable bed (H_M) , in which recirculation is more intensive, smaller values of Ct are expected.

NOTATION

= spout cross section, m²

= annulus cross section, m²

 $a_0 cdots a_4 =$ coefficients in Equation (2), dimensionless $b_0
ldots b_4 = \text{coefficients in Equation (3), dimensionless}$

Ct = Craya-Curtet number D_c = column diameter, m

 d_o = air orifice diameter, m

= equivalent particle diameter (sphere of equal volume), m

= elementary height in the spouted bed, m

 G_{af} , $G_{ai} = air$ throughput in the spout, or in the annulus,

 G_i , G_i = particle throughput in the spout, or in the annulus, kg/s

= radial particle flow rate across the annulus-spout G_r boundary, kg/s

= initial bed height, m

 $H_M = \text{maximum initial spoutable bed height, m}$ $h, h_x =$ level in the bed above the air orifice, m

 i_i = momentum of stream induced by the jet, kg m/s² = momentum of stream passing through the orifice, i_{o} kg m/s²

= coefficient in Equation (1), dimensionless m

= mass throughput of the stream induced by the m_i jet, kg/s

 m_o = mass flow through the orifice, kg/s

= exponent in Equation (1), dimensionless

 Re_p $= d_p v_{pc} \rho_f / \mu = \text{particle Reynolds number}$

= area of a sphere having a volume equal to that of the particle, m²

 $S_{p,ef}$ = effective particle area, m^2

= annular air throughput, Nm3/h

= linear spout air velocity based on empty spout (in the cylindrical part of the spouted bed), m/s

= velocity of the stream induced by the jet, m/s v_i

= velocity of the stream passing through the orifice,

= axial particle velocity, m/s v_p

= arithmetic mean of the average axial particle v_p velocities, at levels h_1 and h_2 , m/s

= free particle falling velocity, m/s v_{pc}

= axial particle velocity in the spout, m/s v_{pf} = axial particle velocity in the annulus, m/s v_{pi}

w = air mass velocity based on empty column, kg/m²s

= minimum mass air velocity required to maintain w_m a stable spouted bed, kg/m²s

Greek Letters

 Δh = axial bed difference in the spouted bed, m

= voidage in the spout €f

= voidage in the annulus €į

= voidage in the bed at the onset of fluidization

= cone angle

= dynamic fluid viscosity, kg/m s

 $= S_p/S_{p,ef} = sphericity$ = fluid density, kg/m³ = particle density, kg/m³

= bulk density, kg/m³

= bulk density in the spout, kg/m³ ρ_{vf}

= bulk density in the annular zone, kg/m³ ρ_{vi}

= average particle residence time, s

= average particle residence time in the conical part of the spouted bed, s

= average particle residence time in the cylindrical part of the annular space, s

particle residence time in the spout, s τ_f

= particle residence time in the outside spout, s Tfe

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